University of BRISTOL

Ultrasonic Arrays 1-
Full Matrix Capture and Imaging Algorithms

Outline

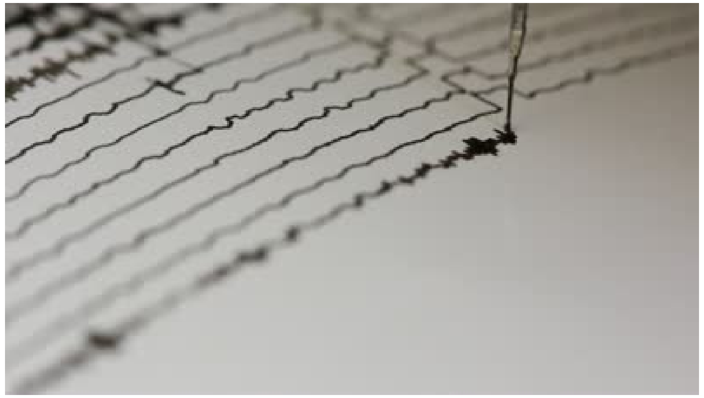
- Introduction
- Modelling classical beam-forming
- Imaging with full matrix array data
- Simulating full matrix array data
- Learning outcomes

Introduction

- It is important to understand the complete process of how an array behaves
  - Focussing on transmission and reception
  - Using this to produce an image
  - What is imaging performance or ideal approach
- How can we simulate this
  - Introduce Full Matrix Capture

Introduction

- Traditional ultrasonic measurements use one or two monolithic* transducers to transmit and receive
- Basic output is a single time-domain signal an A-scan
- A-scan is a 1D function of time (or distance since $z \equiv ct$)



*Sometimes referred to as “single crystal” transducers

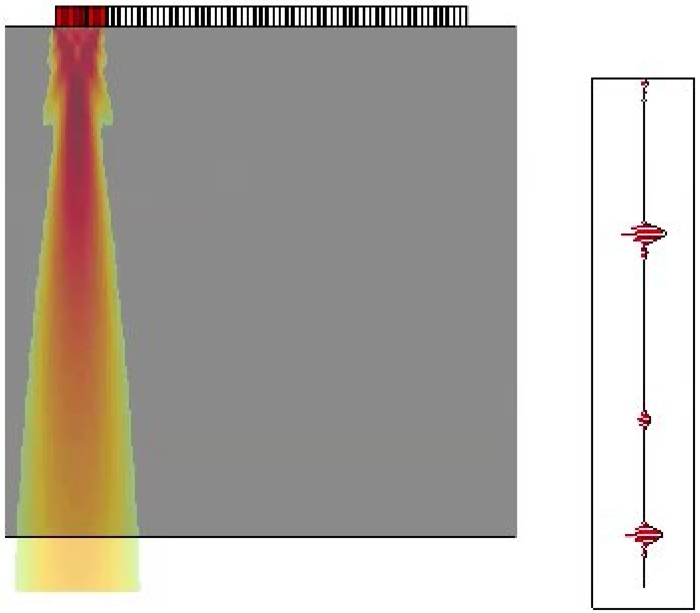
Introduction

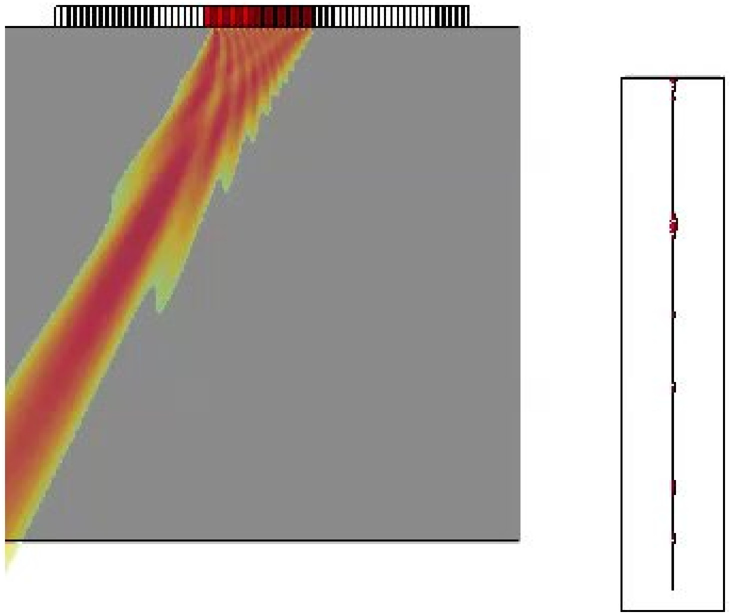
- Monolithic transducer has to be moved to obtain a 2D image from A-scans
  - B-scan – transducer translated (x) or rotated (θ) and A-scans plotted side by side to obtain image as function of (x,z) or (θ,z)
  - C-scan – transducer is raster-scanned in 2D and one parameter (e.g. amplitude) extracted from each A-scan to form image

Introduction

- Originally, arrays just replicated mechanical scanning electronically to obtain B-scan – <u>classical beam-forming</u>

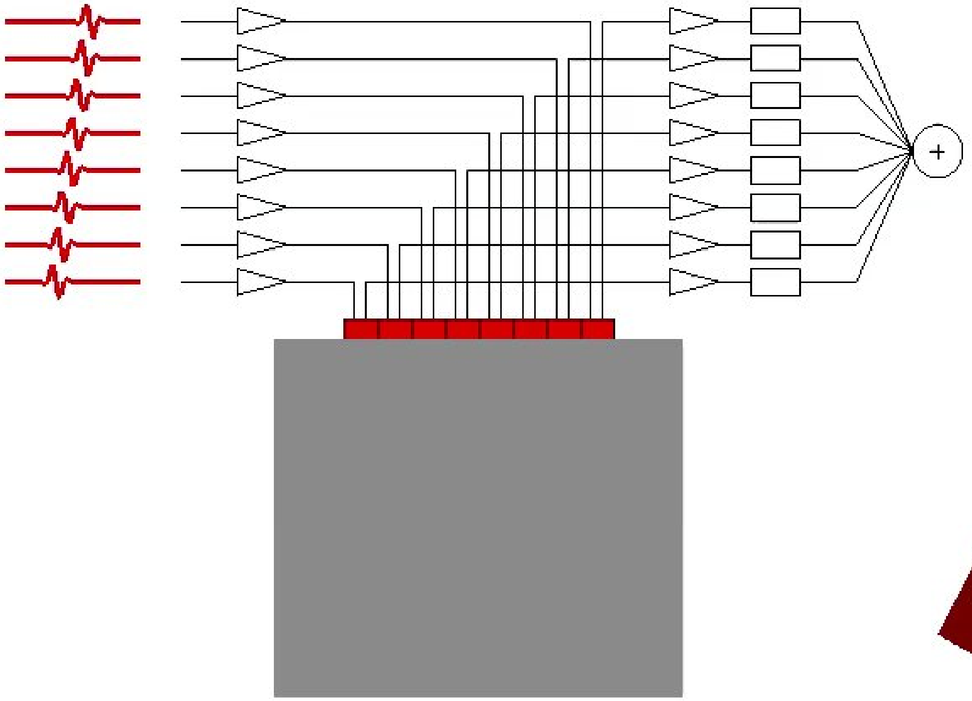
Linear scan

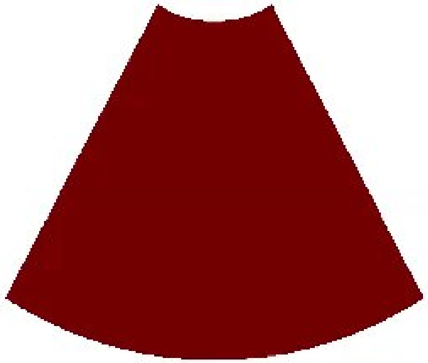




Introduction

- Classical beam-forming – recreates A-scans from monolithic transducer and displays them side-by-side
  - Each <u>transmit cycle</u>

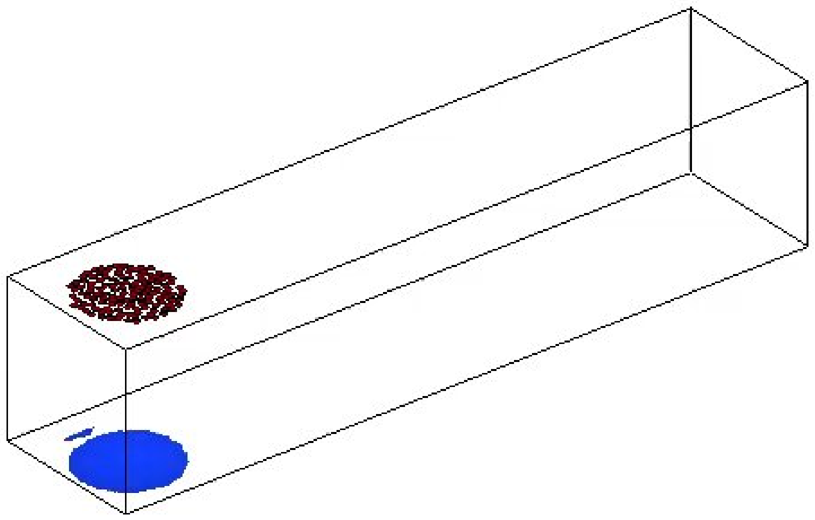




Introduction

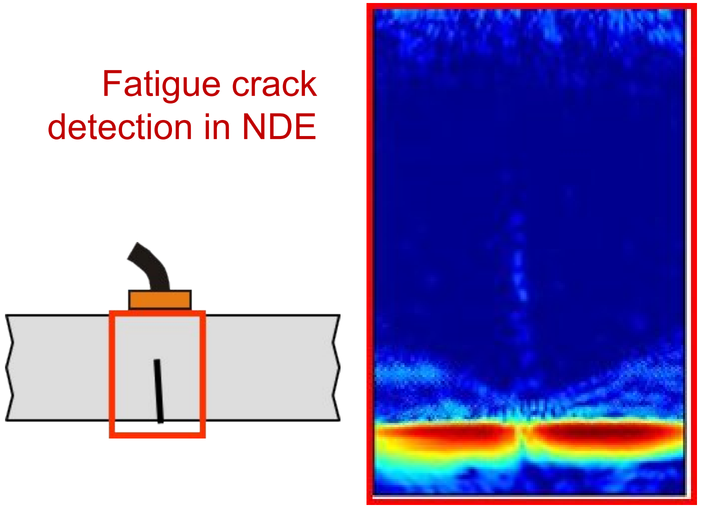
- Examples of ultrasonic array imaging

3D imaging with 2D array in NDE



Foetal imaging in medicine

Fatigue crack detection in NDE



Modelling classical beam-forming

- Objective
  - Predict final image intensity $|I(\mathbf{r})|$ given description of physical "scene" (e.g. location and type of scatterers, component geom.)
- Method (in words)
  1. Predict the A-scan $w_k(t)$ obtained on transmit cycle k
  2. Work out how to map $\{k,t\}$ parameters to position $\mathbf{r}$ in image
  3. Then $I(\mathbf{r}(k,t)) = w_k(t)$ gives one scan-line in image
  4. Repeat for each transmit cycle to get complete image

- Difficult bit is step 1 …

Modelling classical beam-forming

- Predict the A-scan, $w_k(t)$, obtained on transmit cycle k
  - First describe what happens on reception
  - $w_k(t)$ is a weighted sum of delayed received signals, $v_{jk}(t)$, received from each element in the array

$$w_k(t) = \sum_{j=1}^{N} a_{jk}^{(R)} v_{jk}\left(t - \tau_{jk}^{(R)}\right)$$

where $a_{jk}^{(R)}$ and $\tau_{jk}^{(R)}$ are the amplitude and delay for the contribution from the j^{th} receiving element on the k^{th} transmit cycle – these depend on the type of scan

  - Now need to predict $v_{jk}(t)$ …

Modelling classical beam-forming

- Prediction of $v_{jk}(t)$, the signal received from element j on transmit cycle k
  - Linearity of wave equation means we can apply superposition
  - $v_{jk}(t)$ can be written as delayed and weighted sum of contributions from each element transmitting separately with zero delay, $f_{ij}(t)$:

$$v_{jk}(t) = \sum_{i=1}^{N} a_{ik}^{(T)} f_{ij}\left(t - \tau_{ik}^{(T)}\right)$$

where $a_{ik}^{(T)}$ and $\tau_{ik}^{(T)}$ are the amplitude and delay applied to the i^{th} transmitting element on the k^{th} transmit cycle – again these depend on the type of scan

Modelling classical beam-forming

- Combine previous to obtain

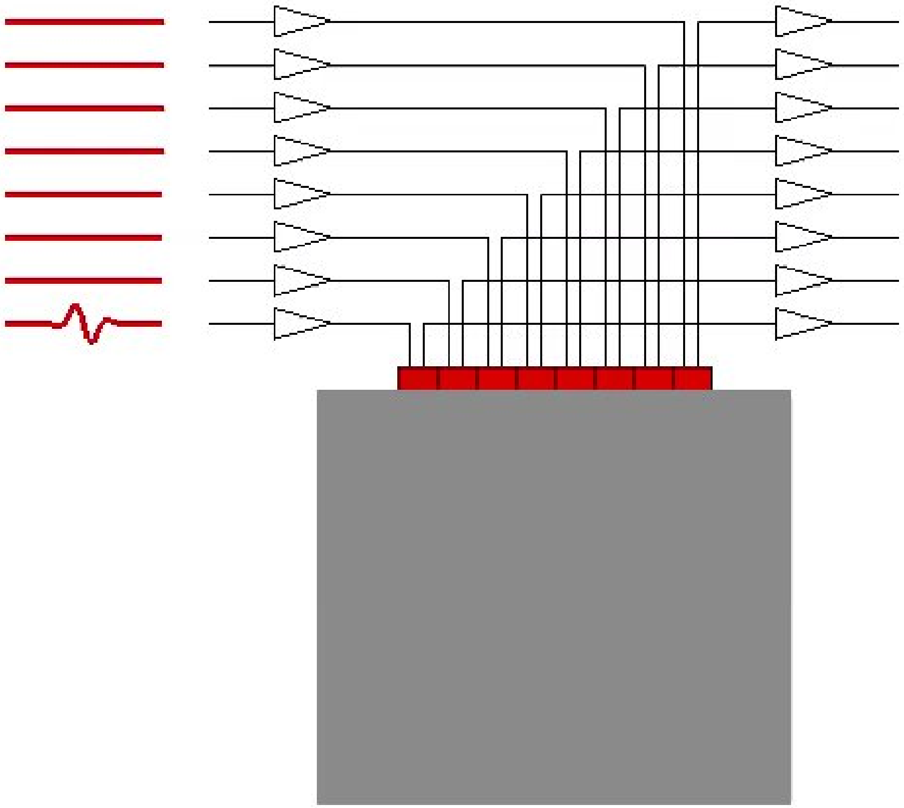
$$I(\mathbf{r}(t,k)) = w_k(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ik}^{(T)} a_{jk}^{(R)} f_{ij}\left(t - \tau_{ik}^{(T)} - \tau_{jk}^{(R)}\right)$$

  – This is an expression for image in terms of most basic unit of array response, $f_{ij}(t)$, the full matrix of array data

- Implications
  – If we can predict $f_{ij}(t)$ then we can simulate any classical beam-forming operation given its weights, $a_{ik}^{(T,R)}$, and delays, $\tau_{ik}^{(T,R)}$
  – But if we physically recorded $f_{ij}(t)$ from a real array we could recreate any classically beam-formed image in post-processing
  – **And we can also develop better imaging algorithms that have no classical beam-forming equivalent**

Imaging with full matrix data

- How it works in practice



Imaging with full matrix data

- More concise version of previous expression

$$I(\mathbf{r}) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(\mathbf{r}) f_{ij}\left(\tau_{ij}(\mathbf{r})\right)$$

where $a_{ij}(\mathbf{r}) = a_{ik(\mathbf{r})}^{(T)} a_{jk(\mathbf{r})}^{(R)}$ and $\tau_{ij}(\mathbf{r}) = t(\mathbf{r}) - \tau_{ik(\mathbf{r})}^{(T)} - \tau_{jk(\mathbf{r})}^{(R)}$

- This can be used to describe almost all linear imaging algorithms

- Also note the slight change in priority
  - Classical beam-forming –a physical A-scan is converted it into a line in the image because $\mathbf{r} = \mathbf{r}(t,k)$
  - In above expression the value at any point in an image can be independently calculated from full matrix of array data because we can write $t = t(\mathbf{r})$ and $k = k(\mathbf{r})$

Imaging with full matrix data

- Examples – linear B-scan

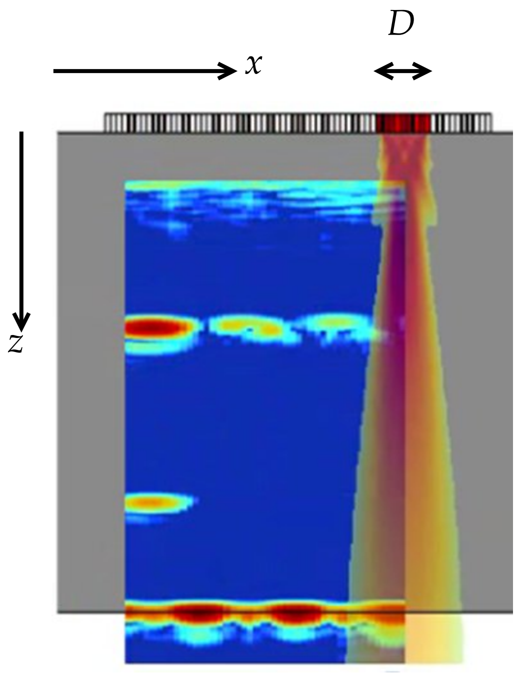
$$I(\mathbf{r}) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}(\mathbf{r}) f_{ij}(\tau_{ij}(\mathbf{r}))$$

If image position $\mathbf{r} = \{x, z\}^T$ then parameters are:

$$a_{ij}(\mathbf{r}) = \begin{cases} 1 & |x - x_i| < \frac{D}{2} \quad \& \quad |x - x_j| < \frac{D}{2} \\ 0 & otherwise \end{cases}$$

$$\tau_{ij}(\mathbf{r}) = 2z/c$$

where D is aperture width, $\mathbf{r_i} = \{x_i, 0\}^T$ is position of i^{th} element in array and c is speed of sound



Imaging with full matrix data

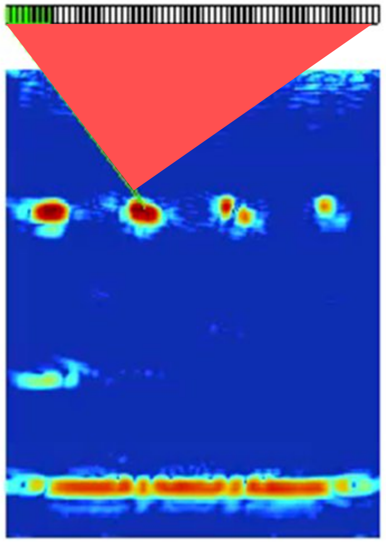
- Example – total focusing method (TFM)
  - Focus whole array at every image point in transmission and reception
  - Close to theoretical image resolution limit (diffraction limit)

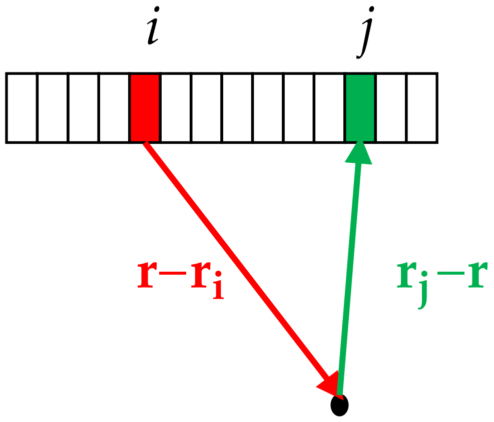
$$I(\mathbf{r}) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}(\mathbf{r}) f_{ij}(\tau_{ij}(\mathbf{r}))$$

Parameters are:

$$a_{ij}(\mathbf{r}) = 1$$

$$\tau_{ij}(\mathbf{r}) = \frac{|\mathbf{r}-\mathbf{r_i}|}{c} + \frac{|\mathbf{r_j}-\mathbf{r}|}{c}$$





Simulating full matrix array data

- Finite element modelling possible
  - Flexible – can model anything
  - Very computationally intensive (need ~10 elements per wavelength and N time-domain simulations per data set)

- Basic principle of ray-based modelling
  - Each signal $f_{ij}(t)$ is a superposition of contributions from different ray-paths
  - Each ray-path is a possible route from transmitter element to receiver
  - The simplest ray-paths are direct reflections from features and defects in the structure

Simulating full matrix array data

- Procedure
    1. Determine ray paths of interest
    2. Simulate signal contribution from each ray path
    3. Sum contributions to obtain final signal

- Notes
    - Part 2 can be performed exactly in frequency-domain or approximately in time-domain

Simulating full matrix array data

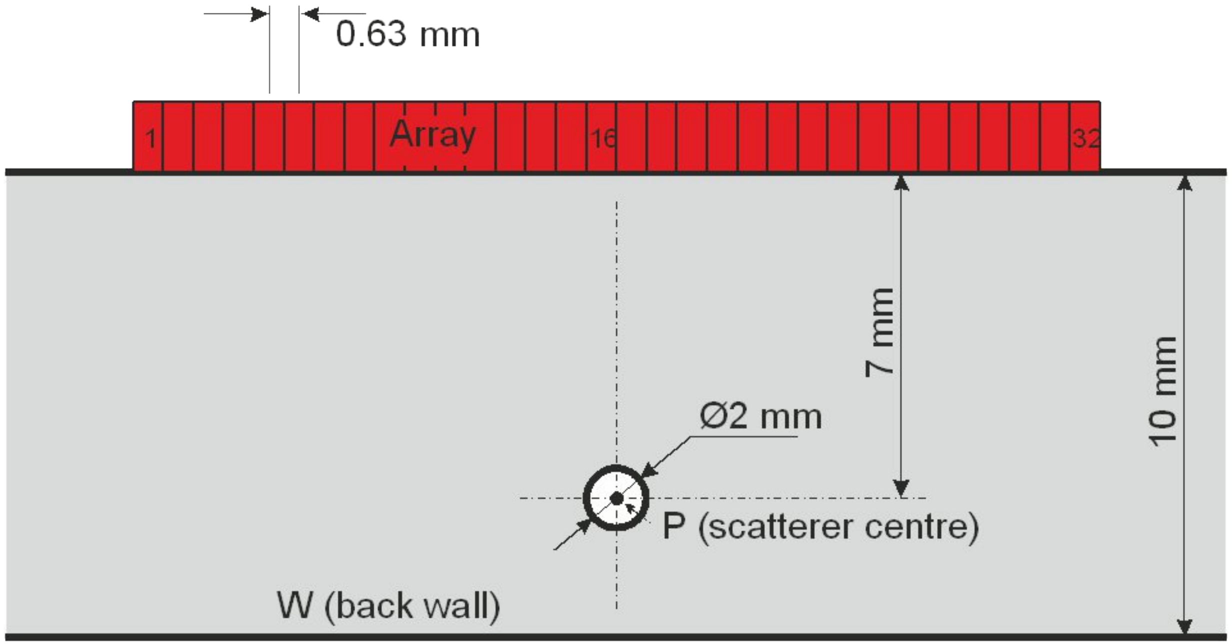
- Freq-domain
  - Simulation of $F_{ij}(\omega)$ in freq-domain
  - Convert to time-domain with inverse Fourier transform

$$f_{ij}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_{ij}(\omega)\exp(i\omega t)d\omega$$

- Time-domain
  - Direct simulation of $f_{ij}(t)$

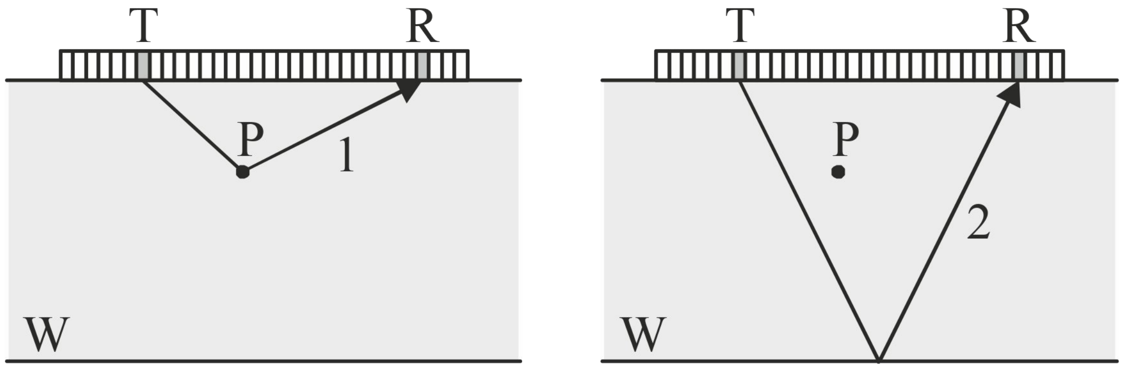
Simulating full matrix array data

- Example 2D geometry – one discrete scatterer (P) and one large planar reflector (W)



Simulating full matrix array data

1. Determine ray-paths of interest for each transmit (T) and receive (R) element combination
   – Example “Direct” reflections from P and W



➢ If both L or S modes were considered there would be 2 cases x 2 legs per case x 2 modes per leg = 8 ray-paths in total

Simulating full matrix array data

2. Simulate signal contribution from each ray path
   - What happens on a typical ray:
     a) INPUT: transmit element excited with electrical pulse (pulse shape)
     b) Pulse leaves transmitting element (element transmit directivity and frequency response)
     c) Pulse propagates along first leg of ray (delay in time, reduction in amplitude due to beam spreading and possibly attenuation)
     d) Pulse is, e.g., reflected off W (reflection coefficient)
     e) Pulse propagates along second leg of ray (...)
     f) Pulse is, e.g., scattered off P (scattering coefficient)
     g) Pulse hits receiving element, R (element receive directivity and frequency response)
     h) OUTPUT: received pulse as a time-domain signal

(what needs to be considered)

Simulating full matrix array data

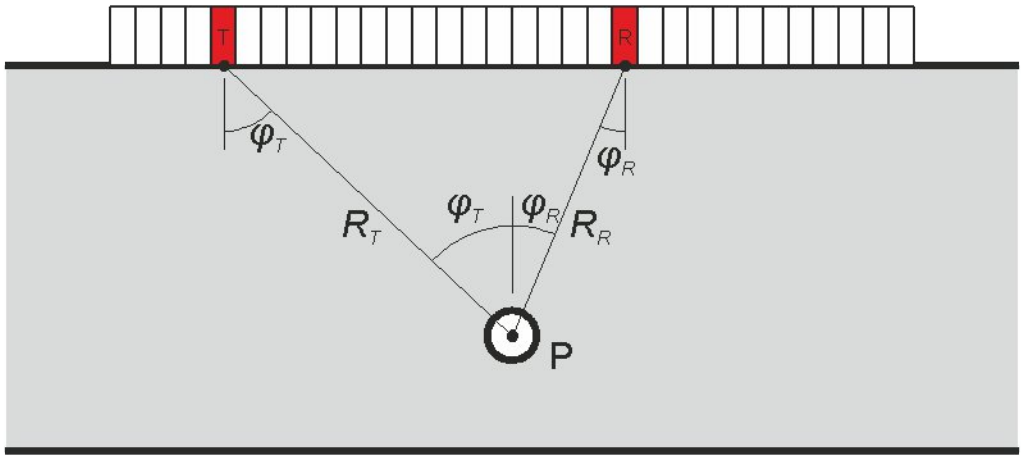
- How to include effects in ray-based model:
  - Input pulse-shape, frequency response of transmitter and frequency response of receiver are lumped into on one parameter: $F_0(\omega)$ in freq. domain or $f_0(t)$ in time domain
  - Element directivity is same in transmission and reception, given by $D_f(\varphi,\omega)$
  - Propagation delay over whole ray-path is $\tau = \sum R_j/c_j$ where R_j and c_j are distance and velocity on each leg and appears as factor of $\exp(-i\omega\tau)$ in freq. domain or $f_0(t-\tau)$ in time domain
  - Beam spread appears as $1/\sqrt{R_T R_R}$ or $1/\sqrt{R_T + R_R}$ factors depending on whether point scatterer or planar reflector between legs 1 and 2
  - Attenuation (if included) appears as $\exp(-\alpha_j R_j)$ factors
  - Also need reflection, $R_c(\theta)$, and scattering coefficients, $S(\varphi_1,\varphi_2;\omega)$

Simulating full matrix array data

– Typical expressions for direct reflections off P

$$F_{ij}(\omega) = F_0(\omega)\frac{1}{\sqrt{R_T R_R}} D_f(\varphi_T,\omega) D_f(\varphi_R,\omega) S(\varphi_T,\varphi_R,\omega) \exp(-i\omega\tau)$$

$$f_{ij}(t) = f_0(t-\tau)\frac{1}{\sqrt{R_T R_R}} D_f(\varphi_T,\omega_c) D_f(\varphi_R,\omega_c) S(\varphi_T,\varphi_R,\omega_c)$$



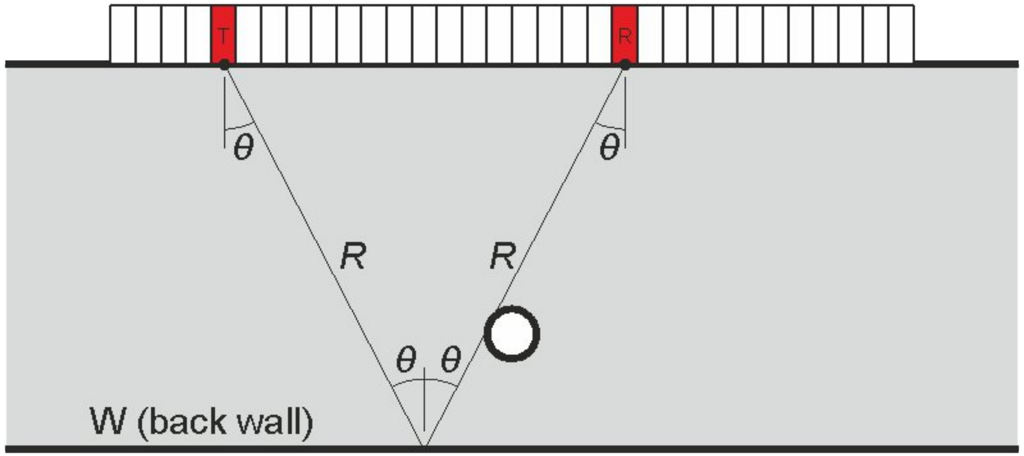
Note that in time-domain, any frequency-dependent parameters f and S must be approximated by their centre-frequency values

Simulating full matrix array data

– Typical expressions for direct reflection off W

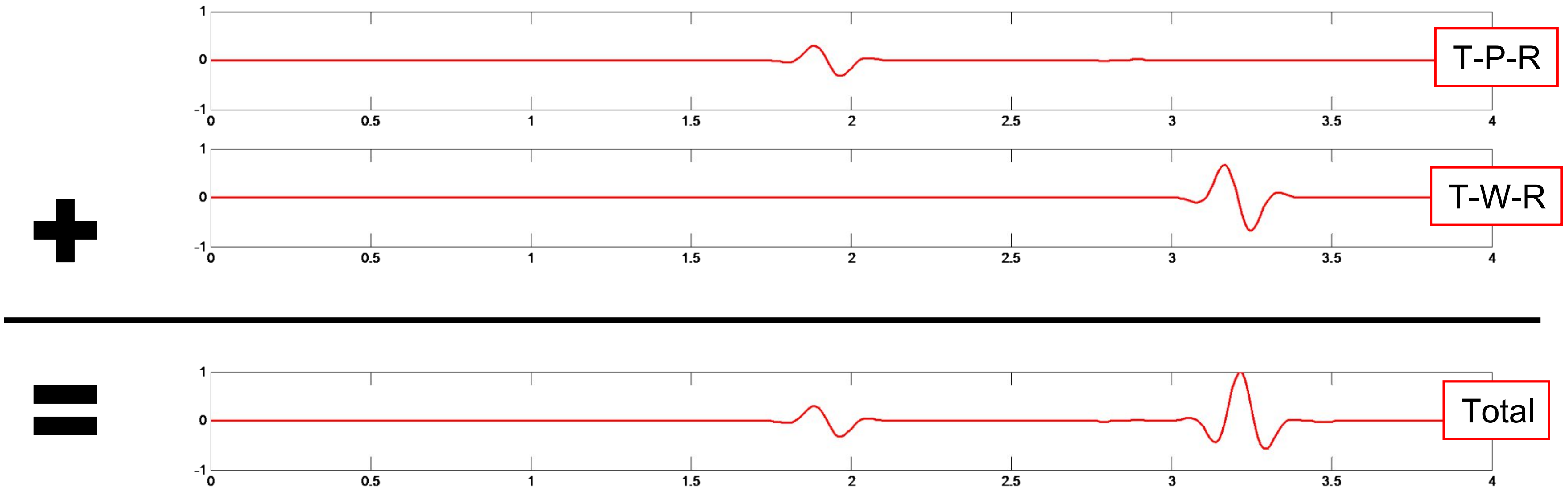
$$F_{ij}(\omega) = F_0(\omega)\frac{1}{\sqrt{R+R}}D_f(\theta,\omega)\,D_f(\theta,\omega)\,R_c(\theta)\exp(-i\omega\tau)$$

$$f_{ij}(t) = f_0(t-\tau)\frac{1}{\sqrt{R+R}}D_f\left(\theta,\omega_c\right)D_f\left(\theta,\omega_c\right)R_c(\theta)$$



Simulating full matrix array data

3. Sum contributions to obtain final signal
   (Can be done in either time or freq-domain)



Learning outcomes

- Understand the relationship between A-scans, B-scans and C-scans
- Know how classical beam-forming is performed with an array
- Know that all imaging algorithms can be written in terms of the full matrix array data
- Understand the basic principles used to simulate the full matrix of array data using a ray-based approach